

# Supersymmetric generalization of the tensor matter fields

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## Abstract

The supersymmetric generalization of a recently proposed Abelian axial gauge model with antisymmetric tensor matter fields is presented.

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# 1 Introduction

A few years ago, a line of investigation has been proposed by Avdeev and Chizhov [1] that consists in treating skew-symmetric rank-2 tensor fields as matter rather than gauge degrees of freedom. The model studied in Ref.[1] has been further reassessed from the point of view of renormalization in the framework of BRS quantization [2]. In view of the potential relevance of matter-like tensor fields for phenomenology [1], it is our purpose in this paper to discuss some facts concerning the formulation of an  $N = 1$  supersymmetric Abelian gauge model realizing the coupling of gauge fields to matter tensor fields and their partners. One intends here to present a superspace formulation of the model and exploit the possible relevance of extra bosonic supersymmetric partners (complex scalars) for the issue of symmetry breaking. We would like to mention that the supersymmetrization of 2-forms that appear as gauge fields is already known in connection with supergravity, and the so-called linear superfields appear to be the most appropriate multiplets to accomodate the 2-form gauge fields [3]. In our case, we aim at the supersymmetrization of matter 2-form fields coupled to Abelian gauge fields and their supersymmetric partners.

The present work is outlined as follows: in Section 2, one searches for the supermultiplet that accomodates the matter tensor field and discusses its self-interaction; the coupling to the gauge supermultiplet is pursued in Sections 3 and 4; in Section 5, one couples the well-known O' Raifeartaigh model [5] to the tensor-field supermultiplet and discusses some features concerning spontaneous symmetry and supersymmetry breaking. Finally, General Conclusions are drawn in Section 6.

## 2 Supersymmetrizing the tensor field

Adopting conventions for the spinor algebra and the superspace parametrization of Ref.[6], one finds that the superfield accomodating the skew-symmetric rank-2 tensor amongst its components is a spinor multiplet subject to the chirality constraint:

$$\begin{aligned} \Sigma_a = & \psi_a + \theta^b \Lambda_{ba} + \theta^2 \mathcal{F}_a - i\theta^c \sigma_{cc}^\mu \bar{\theta}^{\dot{c}} \partial_\mu \psi_a \\ & - i\theta^c \sigma_{cc}^\mu \bar{\theta}^{\dot{c}} \partial_\mu \theta^b \Lambda_{ba} - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu \psi_a, \end{aligned} \quad (2.1)$$

$$\begin{aligned} \bar{\Sigma}_{\dot{a}} = & \bar{\psi}_{\dot{a}} + \bar{\theta}_{\dot{b}} \bar{\Lambda}^{\dot{b}}_{\dot{a}} + \bar{\theta}^2 \bar{\mathcal{F}}_{\dot{a}} + i\theta^c \sigma_{cc}^\mu \bar{\theta}^{\dot{c}} \partial_\mu \bar{\psi}_{\dot{a}} \\ & + i\theta^c \sigma_{cc}^\mu \bar{\theta}^{\dot{c}} \partial_\mu \bar{\theta}_{\dot{b}} \bar{\Lambda}^{\dot{b}}_{\dot{a}} - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu \bar{\psi}_{\dot{a}}, \end{aligned} \quad (2.2)$$

$$\bar{D}_{\dot{b}} \Sigma_a = D_b \bar{\Sigma}_{\dot{a}} = 0, \quad (2.3)$$

where  $\psi_a$  and  $\mathcal{F}_a$  are chiral spinors and  $\Lambda_{ba}$ ,  $\bar{\Lambda}_{\dot{b}\dot{a}}$  are decomposed as:

$$\begin{aligned}\Lambda_{ba} &= \varepsilon_{ba}\rho + \sigma_{ba}^{\mu\nu}\lambda_{\mu\nu} , \\ \bar{\Lambda}_{\dot{b}\dot{a}} &= -\varepsilon_{\dot{b}\dot{a}}\rho^* - \bar{\sigma}_{\dot{b}\dot{a}}^{\mu\nu}\lambda_{\mu\nu}^* .\end{aligned}\tag{2.4}$$

According to the chiral properties of the superfield  $\Sigma_a$ , the  $\lambda_{\mu\nu}$ -tensor corresponds to the  $(1,0)$ -representation of Lorentz group. On the other hand,  $\lambda_{\mu\nu}^*$  yields the  $(0,1)$ -representation. We then write:

$$\begin{aligned}\lambda_{\mu\nu} &= T_{\mu\nu} - i\tilde{T}_{\mu\nu} , \\ \lambda_{\mu\nu}^* &= T_{\mu\nu} + i\tilde{T}_{\mu\nu} ,\end{aligned}\tag{2.5}$$

where  $\tilde{T}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}T^{\alpha\beta}$ . Notice also that  $\tilde{\lambda}_{\mu\nu} = i\lambda_{\mu\nu}$  and  $(\widetilde{\lambda_{\mu\nu}^*}) = -i\lambda_{\mu\nu}^*$ , where the twiddle stands for the dual.

The canonical dimensions of the component fields read as below:

$$\begin{aligned}d(\psi) &= d(\bar{\psi}) = \frac{1}{2} \\ d(\rho) &= d(\lambda_{\mu\nu}) = 1 \\ d(\mathcal{F}) &= d(\bar{\mathcal{F}}) = \frac{3}{2} .\end{aligned}\tag{2.6}$$

Based on dimensional arguments, we propose the following superspace action for the  $\Sigma_a$ -superfield:

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \frac{-1}{32} \{ D^a \Sigma_a \bar{D}_{\dot{a}} \bar{\Sigma}^{\dot{a}} + q \Sigma^a \Sigma_a \bar{\Sigma}_{\dot{a}} \bar{\Sigma}^{\dot{a}} \} .\tag{2.7}$$

To check whether such an action is actually the supersymmetric extension of the model that treats  $T_{\mu\nu}$  as a matter field [1], we have now to write down eq.(2.7) in terms of the component fields  $\psi, \rho, \lambda_{\mu\nu}$  and  $\mathcal{F}$ :

$$\begin{aligned}\mathcal{S} &= \int d^4x \left( + \partial^\mu \rho \partial_\mu \rho^* - 16 \partial^\mu \lambda_{\mu\nu} \partial_\alpha \lambda^{*\alpha\nu} + i \bar{\mathcal{F}}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \mathcal{F}^a - i \bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \partial^\nu \partial_\nu \psi^a \right. \\ &\quad - q \frac{1}{\sqrt{2}} (\rho^2 - 4 \lambda_{\mu\nu} \lambda^{\mu\nu}) \frac{1}{\sqrt{2}} (\rho^{*2} - 4 \lambda_{\alpha\beta}^* \lambda^{*\alpha\beta}) + 4q \lambda^{\mu\nu} \lambda_{\mu\nu} \bar{\mathcal{F}}_{\dot{a}} \bar{\psi}^{\dot{a}} + 4q \lambda^{*\mu\nu} \lambda_{\mu\nu}^* \mathcal{F}^a \psi_a \\ &\quad - 2q \mathcal{F}^a \psi_a \bar{\mathcal{F}}_{\dot{a}} \bar{\psi}^{\dot{a}} - q \rho^2 \bar{\mathcal{F}}_{\dot{a}} \bar{\psi}^{\dot{a}} - q (\rho^*)^2 \mathcal{F}^a \psi_a + \frac{q}{2} \psi^a \psi_a \partial^\mu \partial_\mu (\bar{\psi}_{\dot{a}} \bar{\psi}^{\dot{a}}) \\ &\quad - i q \rho \psi^a \sigma_{a\dot{a}}^\mu \partial_\mu (\bar{\psi}^{\dot{a}} \rho^*) + 4q \rho \psi^a \sigma_{a\dot{a}}^\mu \partial_\beta (\lambda_{\mu}^{*\beta} \bar{\psi}^{\dot{a}}) - 4q \lambda_{\mu\beta} \partial^\beta (\rho^* \bar{\psi}_{\dot{a}}) \bar{\sigma}^{\mu\dot{a}a} \psi_a \\ &\quad \left. - 16iq \lambda_{\mu\alpha} \partial_\beta (\lambda^{*\beta\alpha} \bar{\psi}_{\dot{a}}) \bar{\sigma}^{\mu\dot{a}a} \psi_a \right) .\end{aligned}\tag{2.8}$$

Using  $\lambda_{\mu\nu} = \frac{1}{4}(T_{\mu\nu} - i\tilde{T}_{\mu\nu})$ , we arrive at the relation:

$$16 \partial^\alpha \lambda_{\alpha\mu} \partial_\beta \lambda^{*\beta\mu} = 2 \partial^\alpha T_{\alpha\mu} \partial_\beta T^{\beta\mu} - \frac{1}{2} \partial^\alpha T^{\mu\nu} \partial_\alpha T_{\mu\nu} .\tag{2.9}$$

The action above displays the terms proposed by Avdeev et al. in Ref.[1]; besides the anti-symmetric tensor, there appear a complex scalar and a pair of spinors as its supersymmetric partners:  $\psi_a$ , a non - physical fermion, and  $\mathcal{F}_a$ , corresponding to a physical Weyl spinor. The undesirable presence of a spinor with lower canonical dimension ( $\frac{1}{2}$ , instead of  $\frac{3}{2}$ ) generating, as expected, a higher derivative term in the Lagrangian can be avoided by a reshuffling of the spinorial degrees of freedom, if one keeps the interactions turned off. In fact, one can join both  $\psi_a$  and  $\mathcal{F}_a$  in a single fundamental Dirac spinor,  $\Psi$ , as follows:

$$\Psi = \begin{pmatrix} \mathcal{F}_a(x) \\ \bar{\sigma}^{\mu\dot{a}a} \partial_\mu \psi_a(x) \end{pmatrix}. \quad (2.10)$$

The usual kinetic term,  $i \bar{\Psi} \gamma^\mu \partial_\mu (\Psi)$ , provides the kinetical terms for  $\psi_a$  and  $\mathcal{F}_a$ , turning the higher derivative term,  $-i \bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \partial^2 \psi^a$ , into a matter of choice for the field basis. Nevertheless, this is true only for the free theory. The interaction sector of (2.8) cannot be re-expressed in terms of the Dirac spinor  $\Psi$ , imposing a dissociation back to Weyl spinorial degrees of freedom. Therefore, it happens that the full theory must carry a higher derivative term, giving birth to a conjecture that this might be the fermionic counterpart of problems concerning the transverse sector of the original - bosonic - model for the tensor matter field with interactions, as discussed in [1].

### 3 The gauging of the model

In order to perform the gauging of the model described by eq.(2.7), one proceeds along the usual lines and introduces a chiral scalar superfield,  $\Lambda$ , to act as the gauge parameter:

$$\Lambda = (1 - i\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu) (\phi + \theta^b w_b + \theta^2 \pi) \quad (3.1)$$

$$\bar{\Lambda} = (1 + i\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu) (\phi^* + \bar{\theta}_{\dot{b}} \bar{w}^{\dot{b}} + \bar{\theta}^2 \bar{\pi}). \quad (3.2)$$

The infinitesimal gauge transformations of the superfields  $\Sigma$  and  $\bar{\Sigma}$  read as:

$$\begin{aligned} \delta \Sigma_a &= i h \Lambda \Sigma_a \\ \delta \bar{\Sigma}_{\dot{a}} &= -i h \bar{\Lambda} \bar{\Sigma}_{\dot{a}}, \end{aligned} \quad (3.3)$$

and the behaviour of  $(D^a \Sigma_a)$  and  $(\bar{D}_{\dot{a}} \bar{\Sigma}^{\dot{a}})$  under finite transformations become:

$$\begin{aligned} D^a \Sigma'_a &= e^{i h \Lambda} (D^a \Sigma_a + i h D^a \Lambda \Sigma_a) \\ \bar{D}_{\dot{a}} \bar{\Sigma}'^{\dot{a}} &= e^{-i h \bar{\Lambda}} (\bar{D}_{\dot{a}} \bar{\Sigma}^{\dot{a}} - i h \bar{D}_{\dot{a}} \bar{\Lambda} \bar{\Sigma}^{\dot{a}}). \end{aligned} \quad (3.4)$$

To gauge-covariantize the superspace derivatives, one introduces a gauge connection superfield:

$$D_a \rightarrow \nabla_a = D_a + i h \Gamma_a, \quad (3.5)$$

in such a way that  $\Gamma_a$  transforms like

$$\Gamma'_a = \Gamma_a - D_a \Lambda. \quad (3.6)$$

This yields:

$$(\nabla^a \Sigma_a)' = e^{ih\Lambda} (\nabla^a \Sigma_a). \quad (3.7)$$

To achieve a U(1)-invariant action, one proposes

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \left( \nabla^a \Sigma_a e^{hV} \overline{\nabla_{\dot{a}} \Sigma^{\dot{a}}} \right), \quad (3.8)$$

where  $V$  is the real scalar superfield [9] that accomplishes the gauging of supersymmetric QED [10]:

$$V' = V + i(\bar{\Lambda} - \Lambda). \quad (3.9)$$

At this point, the gauge sector displays more degrees of freedom than what is actually required to perform the gauging. There are component vector fields in both  $\Gamma_a$  and  $V$ . However, we notice that the superfield  $\Gamma_a$  is not a true independent gauge potential. Indeed,

$$\Gamma_a = -iD_a V \quad (3.10)$$

reproduces correctly the gauge transformation of  $\Gamma_a$  and, at the same time, eliminates the redundant degrees of freedom that would be otherwise present, if we were to keep  $\Gamma_a$  and  $V$  as gauge superfields. Therefore, the locally U(1)-invariant action takes over the form:

$$\begin{aligned} \mathcal{S} = & \int d^4x d^2\theta \frac{-1}{128} \left( \bar{D}^2 (e^{-V} D^a e^V) \bar{D}^2 (e^{-V} D_a e^V) \right) \\ & + \int d^4x d^2\theta d^2\bar{\theta} \frac{-1}{32} \left( \nabla^a \Sigma_a e^{hV} \overline{\nabla_{\dot{a}} \Sigma^{\dot{a}}} + q \Sigma^a \Sigma_a e^{2hV} \overline{\Sigma_{\dot{a}} \Sigma^{\dot{a}}} \right), \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} \nabla^a \Sigma_a &= D^a \Sigma_a + h D^a V \Sigma_a \\ \overline{\nabla_{\dot{a}} \Sigma^{\dot{a}}} &= \bar{D}_{\dot{a}} \bar{\Sigma}^{\dot{a}} + h \bar{D}_{\dot{a}} V \bar{\Sigma}^{\dot{a}}. \end{aligned} \quad (3.12)$$

The  $\theta$ -expansion for the superfield  $V$  brings about the following component fields:

$$\begin{aligned} V = & C + \theta^a b_a + \bar{\theta}_{\dot{a}} \bar{b}^{\dot{a}} + \theta^a \bar{\theta}^{\dot{a}} \sigma_{a\dot{a}}^\mu A_\mu \\ & + \theta^2 \lambda + \bar{\theta}^2 \bar{\lambda} + \theta^2 \bar{\theta}_{\dot{a}} \bar{\gamma}^{\dot{a}} + \bar{\theta}^2 \theta^a \gamma_a + \theta^2 \bar{\theta}^2 \Delta, \end{aligned} \quad (3.13)$$

where  $C$ ,  $\lambda$ ,  $\bar{\lambda}$  and  $\Delta$  are scalars  $b_a$  and  $\gamma_a$  are spinors and  $A_\mu$  is the U(1)-gauge field. The gauge transformation of these fields read as below:

$$\begin{aligned} \delta C &= i(\phi^* - \phi), \quad \delta \lambda = -i\pi, \quad \delta \bar{\lambda} = i\bar{\pi}, \\ \delta b_a &= -i w_a, \quad \delta \bar{b}_{\dot{a}} = i \bar{w}_{\dot{a}} \\ \delta \Delta &= \frac{i}{4} \partial^\mu \partial_\mu (\phi - \phi^*), \quad \delta A^\mu = -\partial^\mu (\phi + \phi^*), \\ \delta \gamma_a &= \frac{1}{2} \sigma_{a\dot{a}}^\mu \partial_\mu \bar{w}^{\dot{a}}, \quad \delta \bar{\gamma}_{\dot{a}} = -\frac{1}{2} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu w^a. \end{aligned} \quad (3.14)$$

As already known, for the sake of component-field calculations, one usually works in the so-called Wess-Zumino gauge, where  $C$ ,  $b_a$  and  $\lambda$  are gauged away. The expansion for the exponential of the gauge superfield simplifies, in this gauge, according to:

$$e^{hV} = 1 + h\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} A_\mu + h\theta^2 \bar{\theta}_{\dot{a}} \bar{\gamma}^{\dot{a}} + h\bar{\theta}^2 \theta^a \gamma_a + h\theta^2 \bar{\theta}^2 \Delta + \frac{1}{4} h^2 \theta^2 \bar{\theta}^2 A^\mu A_\mu. \quad (3.15)$$

Using this gauge, the transformations of the matter fields are:

$$\psi_a = ih\phi\psi_a, \quad \delta\rho = ih\phi\rho, \quad \delta\lambda_{\mu\nu} = ih\phi\lambda_{\mu\nu}, \quad \delta\mathcal{F}_a = ih(\phi\mathcal{F}_a); \quad (3.16)$$

we get thereby the following transformations for the components  $T_{\mu\nu}$  and  $\tilde{T}_{\mu\nu}$ :

$$\delta T_{\mu\nu} = h\phi\tilde{T}_{\mu\nu}, \quad \delta\tilde{T}_{\mu\nu} = -h\phi T_{\mu\nu} \quad (3.17)$$

These are precisely the Abelian gauge transformations for the tensor field as firstly proposed in Ref.[1].

## 4 Component-field action in the Wess-Zumino gauge

Having adopted the component fields as defined in the previous sections, lengthy algebraic computations yield the following action in the Wess-Zumino gauge:

$$\begin{aligned} \mathcal{S} = \int d^4x \bigg( & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + 2\Delta^2 + i\bar{\gamma}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \gamma^a + \partial^\mu \rho \partial_\mu \rho^* - 16\partial^\mu \lambda_{\mu\nu} \partial_\alpha \lambda^{*\alpha\nu} \\ & + i\bar{\mathcal{F}}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \mathcal{F}^a - i\bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \partial_\nu \psi^a - i\frac{h}{2} \partial^\mu \rho A_\mu \rho^* + i\frac{h}{2} \rho A^\mu \partial_\mu \rho^* \\ & + 2h\rho\Delta\rho^* + \frac{h^2}{4} \rho A^\mu A_\mu \rho^* - h\gamma^a \mathcal{F}_a \rho^* - h\rho \bar{\gamma}_{\dot{a}} \bar{\mathcal{F}}^{\dot{a}} - i\frac{h}{2} \rho \gamma^a \sigma_{a\dot{a}}^\mu \partial_\mu \bar{\psi}^{\dot{a}} \\ & - i\frac{h}{2} \rho^* \bar{\gamma}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \psi^a + i\frac{h}{2} \partial_\mu \rho^* \psi^a \sigma_{a\dot{a}}^\mu \bar{\gamma}^{\dot{a}} - i\frac{h}{2} \partial_\mu \rho \gamma^a \sigma_{a\dot{a}}^\mu \bar{\psi}^{\dot{a}} - \frac{h}{4} \psi^a \sigma_{a\dot{a}}^\mu A_\mu \partial_\nu \bar{\psi}^{\dot{a}} \\ & + \frac{h}{4} \bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu A_\mu \partial_\nu \partial_\nu \psi^a + ih\Delta \bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \psi^a + ih\Delta \psi^a \sigma_{a\dot{a}}^\mu \partial_\mu \bar{\psi}^{\dot{a}} - \frac{h}{4} \partial^\nu A_\nu \bar{\psi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \psi^a \\ & + \frac{h}{4} \partial_\nu A^\nu \psi^a \sigma_{a\dot{a}}^\mu \partial_\mu \bar{\psi}^{\dot{a}} + \frac{h}{2} \mathcal{F}^a \sigma_{a\dot{a}}^\mu A_\mu \bar{\mathcal{F}}^{\dot{a}} - \frac{h}{8} \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} F_{\mu\nu} \partial_\alpha \bar{\psi}^{\dot{a}} \\ & + \frac{h}{8} \partial_\mu \psi^a (\sigma^\mu \bar{\sigma}^\alpha \sigma^\nu)_{a\dot{a}} F_{\nu\alpha} \bar{\psi}^{\dot{a}} + h\rho^* F_{\mu\nu} \lambda^{\mu\nu} + h\rho F_{\mu\nu} \lambda^{*\mu\nu} \\ & + 2ih(4\partial_\mu \lambda^{*\mu\nu} \lambda_{\nu\alpha} A^\alpha - 4\partial^\mu \lambda_{\mu\nu} A_\alpha \lambda^{*\nu\alpha}) \\ & - i\frac{h^2}{2} \gamma^a (\sigma^\mu \bar{\sigma}^\nu \sigma^\alpha)_{a\dot{a}} \lambda_{\mu\nu} A_\alpha \bar{\psi}^{\dot{a}} + \frac{h}{2} \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} \bar{\gamma}^{\dot{a}} \partial_\nu \lambda_{\mu\alpha}^* - \frac{h}{2} \partial_\nu \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} \lambda_{\mu\alpha}^* \bar{\gamma}^{\dot{a}} \\ & + i\frac{h^2}{2} \bar{\gamma}^{\dot{a}} (\bar{\sigma}^\mu \sigma^\alpha \bar{\sigma}^\beta)_{\dot{a}a} A_\mu \psi^a \lambda_{\alpha\beta}^* - \frac{h}{2} \gamma^a (\sigma^\alpha \bar{\sigma}^\beta \sigma^\mu)_{a\dot{a}} \partial_\mu \lambda_{\alpha\beta} \bar{\psi}^{\dot{a}} + \frac{h}{2} \gamma^a (\sigma^a \bar{\sigma}^\beta \sigma^\mu)_{a\dot{a}} \lambda_{\alpha\beta} \partial_\mu \bar{\psi}^{\dot{a}} \\ & + \frac{h}{2} \partial_\mu \psi^a (\sigma^\mu \bar{\sigma}^\nu \sigma^\alpha)_{a\dot{a}} A_\nu \partial_\alpha \bar{\psi}^{\dot{a}} + \frac{h}{4} \partial_\mu \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} A_\nu \partial_\alpha \bar{\psi}^{\dot{a}} \\ & + \frac{h}{4} \partial_\mu \psi^a (\sigma^\mu \bar{\sigma}^\alpha \sigma^\nu)_{a\dot{a}} A_\nu \partial_\alpha \bar{\psi}^{\dot{a}} + h^2 \Delta \psi^a \sigma_{a\dot{a}}^\mu A_\mu \bar{\psi}^{\dot{a}} + \frac{i}{16} h^2 \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} F_{\mu\nu} A_\alpha \bar{\psi}^{\dot{a}} \\ & + i\frac{h^2}{16} \psi^a (\sigma^\nu \bar{\sigma}^\mu \sigma^\alpha)_{a\dot{a}} F_{\alpha\mu} A_\nu \bar{\psi}^{\dot{a}} - i\frac{h^2}{2} \gamma^a (\sigma^\alpha \bar{\sigma}^\beta \sigma^\mu)_{a\dot{a}} \lambda_{\alpha\beta} A_\mu \bar{\psi}^{\dot{a}} + \end{aligned}$$

$$\begin{aligned}
& -i\frac{h^2}{4}\partial_\mu\psi^a(\sigma^\mu\bar{\sigma}^\nu\sigma^\alpha)_{a\dot{a}}A_\nu A_\alpha\bar{\psi}^{\dot{a}} - i\frac{h^2}{8}\partial_\mu\psi^a(\sigma^\nu\bar{\sigma}^\mu\sigma^\alpha)_{a\dot{a}}A_\nu A_\alpha\bar{\psi}^{\dot{a}} \\
& +i\frac{h^2}{4}\psi^a A_\nu A_\mu(\sigma^\nu\bar{\sigma}^\mu\sigma^\alpha)_{a\dot{a}}\partial_\alpha\bar{\psi}^{\dot{a}} + i\frac{h^2}{8}\psi^a A_\nu A_\mu(\sigma^\nu\bar{\sigma}^\alpha\sigma^\mu)_{a\dot{a}}\partial_\alpha\bar{\psi}^{\dot{a}} \\
& +\frac{h^3}{8}\psi^a(\sigma^\nu\bar{\sigma}^\alpha\sigma^\mu)_{a\dot{a}}A_\nu A_\alpha A_\mu\bar{\psi}^{\dot{a}} - h^2\gamma^a\psi_a\bar{\gamma}_{\dot{a}}\bar{\psi}^{\dot{a}} + 4h^2 A^\nu A_\mu\lambda_{\nu\beta}\lambda^{*\beta\mu} \\
& -q\frac{1}{\sqrt{2}}(\rho^2 - 4\lambda_{\mu\nu}\lambda^{\mu\nu})\frac{1}{\sqrt{2}}(\rho^{*2} - 4\lambda_{\alpha\beta}^*\lambda^{*\alpha\beta}) + 4q\lambda^{\mu\nu}\lambda_{\mu\nu}\bar{\mathcal{F}}_{\dot{a}}\bar{\psi}^{\dot{a}} + 4q\lambda^{*\mu\nu}\lambda_{\mu\nu}^*\mathcal{F}^a\psi_a \\
& -2q\mathcal{F}^a\psi_a\bar{\mathcal{F}}_{\dot{a}}\bar{\psi}^{\dot{a}} - q\rho^2\bar{\mathcal{F}}_{\dot{a}}\bar{\psi}^{\dot{a}} - q(\rho^*)^2\mathcal{F}^a\psi_a + \frac{q}{2}\psi^a\psi_a\partial^\mu\partial_\mu(\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}}) \\
& -iq\rho\psi^a\sigma_{a\dot{a}}^\mu\partial_\mu(\bar{\psi}^{\dot{a}}\rho^*) + 4q\rho\psi^a\sigma_{a\dot{a}}^\mu\partial^\beta(\lambda_{\beta\mu}^*\bar{\psi}^{\dot{a}}) - 4q\lambda_{\mu\beta}\partial^\beta(\rho^*\bar{\psi}_{\dot{a}})\bar{\sigma}^{\mu\dot{a}a}\psi_a \\
& -16iq\lambda_{\mu\alpha}\partial_\beta(\lambda^{*\beta\alpha}\bar{\psi}_{\dot{a}})\bar{\sigma}^{\mu\dot{a}a}\psi_a - qh\Delta\psi^a\psi_a\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}} - iq\frac{h}{2}A_\mu\psi^a\psi_a\partial^\mu(\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}}) \\
& +iq\frac{h}{2}A_\mu\partial^\mu(\psi^a\psi_a)\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}} - q\frac{h^2}{2}A^\mu A_\mu\psi^a\psi_a\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}} - qh\rho\gamma^a\psi_a\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}} - qh\rho^*\bar{\gamma}_{\dot{a}}\bar{\psi}^{\dot{a}}\psi^a\psi_a \\
& -qh\gamma^a\sigma_{ab}^{\mu\nu}\psi^b\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}}\lambda_{\mu\nu} + qh\bar{\gamma}_{\dot{a}}\bar{\sigma}_{\dot{a}\dot{b}}^{\mu\nu}\lambda_{\mu\nu}^*\bar{\psi}^{\dot{b}}\psi^a\psi_a - qh\rho\psi^a\sigma_{a\dot{a}}^\mu A_\mu\bar{\psi}^{\dot{a}}\rho^* \\
& -2iqh\rho\psi^a A_\mu(\sigma^\mu\bar{\sigma}^\alpha\sigma^\beta)_{a\dot{a}}\lambda_{\alpha\beta}^*\bar{\psi}^{\dot{a}} + 2iqh\rho^*\psi^a(\sigma^\beta\bar{\sigma}^\alpha\sigma^\mu)_{a\dot{a}}A_\mu\lambda_{\alpha\beta}\bar{\psi}^{\dot{a}} \\
& +2qh\psi^a(\sigma^\alpha\bar{\sigma}^\beta\sigma^\gamma\bar{\sigma}^\mu\sigma^\nu)_{a\dot{a}}\lambda_{\alpha\beta}A_\gamma\lambda_{\mu\nu}^*\bar{\psi}^{\dot{a}}.
\end{aligned} \tag{4.1}$$

We should stress here a remarkable difference with respect to the case of the chiral and anti-chiral scalar superfields (Wess-Zumino model [4]), namely, the minimal coupling of  $\Sigma$  and  $\bar{\Sigma}$  to the gauge sector necessarily affects the  $\Sigma$  - superfield self-interaction terms as one reads off from eq (3.11). The gauging of the  $U(1)$  - symmetry enriches the self-interactions of the tensor field not only through its fermionic supersymmetric partners, but also through the introduction of the gauge boson and the gaugino at the level of the matter self-interaction terms (these new couplings are compatible with power-counting renormalizability). This is so because the model presented here is based on a single spinor superfield. Had we introduced a couple of spinor superfields,  $\Sigma_a$  and  $\mathcal{T}_a$ , with opposite  $U(1)$  charges:

$$\begin{aligned}
\Sigma'_a &= e^{ih\Lambda}\Sigma_a, \\
\mathcal{T}'_a &= e^{-ih\Lambda}\mathcal{T}_a,
\end{aligned} \tag{4.2}$$

a self-interacting term of the form  $(\Sigma^a\mathcal{T}_a\bar{\Sigma}_{\dot{a}}\bar{\mathcal{T}}^{\dot{a}})$  would automatically be invariant whenever the symmetry is gauged, and there would be no need for introducing the vector superfield to ensure local invariance. Such a mixed self-interacting term could, in principle, be thought of as a possible source for a mass term for the spinor superfields, whenever the physical scalar component  $\rho$  develops a non-trivial vacuum expectation value. Nevertheless, by analysing the  $\rho$  - field interactions in the scalar potential, one concludes that there is no room for spontaneous symmetry breaking as induced by such a component field (and, similarly, for its counterpart inside  $\mathcal{T}$ ). On the other hand, we could think to introduce a gauge-invariant mass term of the form

$$\mathcal{S}_{mass} = \int d^4x (d^2\theta \ i\frac{m}{16}\Sigma^a\mathcal{T}_a - d^2\bar{\theta} \ i\frac{m}{16}\bar{\Sigma}_{\dot{a}}\bar{\mathcal{T}}^{\dot{a}}); \tag{4.3}$$

however, a mixed mass term like the one above introduces two massive excitations of the type  $k^4 = m^4$  in the spectrum. So, regardless the sign of  $m^2$ , a tachyon shall always be present; hence such a mass term is disregarded.

## 5 On Spontaneous Symmetry Breaking

Due to the spinorial character of the superfield  $\Sigma_a$ , it cannot be used to accomplish a spontaneous breaking. Indeed, Lorentz invariance is lost whenever  $\Sigma_a$  acquires a non-trivial vacuum expectation value. The idea in the present section is to couple, in a gauge-invariant manner, the well-known O' Raifeartaigh model [5] to the spinor superfield  $\Sigma_a$ , so as to understand the issue of an eventual mass generation for  $\Sigma_a$  via spontaneous internal symmetry or supersymmetry breakingdown. For the sake of concreteness, the model we adopt to discuss such a matter is defined by the action below:

$$\begin{aligned}
\mathcal{S} = & \int d^4x d^2\theta d^2\bar{\theta} \left( \bar{\phi}\phi + \bar{\phi}_+ e^{hV} \phi_+ + \bar{\phi}_- e^{-hV} \phi_- \right) \\
& + \int d^4x d^2\theta \left( \frac{1}{2} m \phi^2 + \mu \phi_+ \phi_- + f \phi + g \phi \phi_+ \phi_- + G \Sigma^a \Sigma_a \phi_- \phi_- \right) \\
& + \int d^4x d^2\bar{\theta} \left( \frac{1}{2} m \bar{\phi}^2 + \mu \bar{\phi}_+ \bar{\phi}_- + f \bar{\phi} + g \bar{\phi} \bar{\phi}_+ \bar{\phi}_- + G \bar{\Sigma}_{\dot{a}} \bar{\Sigma}^{\dot{a}} \bar{\phi}_- \bar{\phi}_- \right) \\
& + \int d^4x d^2\theta \frac{-1}{128} \left( \bar{D}^2 (e^{-V} D^a e^V) \bar{D}^2 (e^{-V} D_a e^V) \right) \\
& + \int d^4x d^2\theta d^2\bar{\theta} \frac{-1}{32} \left( \nabla^a \Sigma_a e^{hV} \bar{\nabla}_{\dot{a}} \bar{\Sigma}^{\dot{a}} + q \Sigma^a \Sigma_a e^{2hV} \bar{\Sigma}_{\dot{a}} \bar{\Sigma}^{\dot{a}} \right),
\end{aligned} \tag{5.1}$$

where the chiral scalar superfields  $\phi$ ,  $\phi_+$  and  $\phi_-$  are parametrized as follows:

$$\begin{aligned}
\phi &= (1 - i\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu) (A + \theta^b \xi_b + \theta^2 b) \\
\phi_+ &= (1 - i\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu) (A_+ + \theta^b \xi_{+b} + \theta^2 b_+) \\
\phi_- &= (1 - i\theta^a \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu) (A_- + \theta^b \xi_{-b} + \theta^2 b_-);
\end{aligned} \tag{5.2}$$

$m$  and  $\mu$  are mass parameters,  $f$  has dimension of  $mass^2$ , whereas  $g$  and  $G$  are dimensionless coupling constants.  $\Sigma_a$  and  $\phi_-$  have opposite  $U(1)$  - charges. This action, in terms of components, reads:

$$\begin{aligned}
\mathcal{S} = & \int d^4x \left( 4 \{ \partial^\mu A^* \partial_\mu A + \partial^\mu A_-^* \partial_\mu A_- + \partial^\mu A_+^* \partial_\mu A_+ \} + 4 \{ b^* b + b_-^* b_- + b_+^* b_+ \} \right. \\
& \left. + 2i \{ \bar{\xi}^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \xi^a + \bar{\xi}_-^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \xi_-^a + \bar{\xi}_+^{\dot{a}} \bar{\sigma}_{\dot{a}a}^\mu \partial_\mu \xi_+^a \} \right) +
\end{aligned}$$



$$\begin{aligned}
& + \int d^4x \left( 16hA_+^* \Delta A_+ + 8ihA_+^* \partial^\mu A_\mu A_+ + 4h^2 A_+^* A^\mu A_\mu A_+ - 8hA_+^* \gamma^a \xi_{+a} \right. \\
& \left. - 8h\bar{\xi}_{+\dot{a}} \bar{\gamma}^{\dot{a}} A_+ + 4h\xi^{+a} \sigma_{a\dot{a}}^\mu A_\mu \bar{\xi}_{+\dot{a}} + 16ih\partial^\mu A_+^* A_\mu A_+ \right) \\
& + \int d^4x \left( -16hA_-^* \Delta A_- - 8ihA_-^* \partial^\mu A_\mu A_- + 4h^2 A_-^* A^\mu A_\mu A_- + 8hA_-^* \gamma^a \xi_{-a} \right. \\
& \left. + 8h\bar{\xi}_{-\dot{a}} \bar{\gamma}^{\dot{a}} A_- - 4h\xi^{-a} \sigma_{a\dot{a}}^\mu A_\mu \bar{\xi}_{-\dot{a}} - 16ih\partial^\mu A_-^* A_\mu A_- \right) \\
& + \int d^4x \left( m(-4bA + \xi^a \xi_a) + m(-4b^* A^* + \bar{\xi}_{\dot{a}} \bar{\xi}^{\dot{a}}) - 4fb - 4fb^* \right) \\
& + \int d^4x 2 \left( \mu(-2b_+ A_- - 2b_- A_+ + \xi_+^a \xi_{a-}) + \mu(-2b_+^* A_-^* - 2b_-^* A_+^* + \bar{\xi}_{\dot{a}+} \bar{\xi}_{-}^{\dot{a}}) \right) \\
& + \int d^4x 2 \left( g(-2bA_+ A_- - 2Ab_+ A_- - 2AA_+ b_- + A\xi_+^a \xi_{a-} + A_- \xi^a \xi_{a+} + A_+ \xi^a \xi_{a-}) \right. \\
& \left. + g(-2b^* A_+^* A_-^* - 2A^* b_+^* A_-^* - 2A^* A_+^* b_-^* + A^* \bar{\xi}_{\dot{a}+} \bar{\xi}_{-}^{\dot{a}} + A_-^* \bar{\xi}_{\dot{a}} \bar{\xi}_{+}^{\dot{a}} + A_+^* \bar{\xi}_{\dot{a}} \bar{\xi}_{-}^{\dot{a}}) \right) \\
& + \int d^4x 4G \left( (-2\mathcal{F}^a \psi_a + \rho^2 + 4\lambda_{\mu\nu} \lambda^{\mu\nu})(A_-)^2 + (-2\bar{\mathcal{F}}_{\dot{a}} \bar{\psi}^{\dot{a}} + (\rho^*)^2 + 4\lambda_{\mu\nu}^* \lambda^{*\mu\nu})(A_-^*)^2 \right. \\
& + 2\psi^b \psi_b (-4b_- A_- + \xi_-^a \xi_{a-}) + 2\bar{\psi}_{\dot{b}} \bar{\psi}^{\dot{b}} (-4b_-^* A_-^* + \bar{\xi}_{\dot{a}-} \bar{\xi}_{-}^{\dot{a}}) \\
& \left. - 2(\rho\psi^b - \sigma^{\mu\nu ba} \lambda_{\mu\nu} \psi_a)(\xi_{b-} A_-) + 2(\rho^* \bar{\psi}_{\dot{b}} - \lambda_{\mu\nu}^* \bar{\sigma}_{\dot{b}\dot{a}}^{\mu\nu} \bar{\psi}^{\dot{a}})(\bar{\xi}_{-}^{\dot{b}} A_-^*) \right) \\
& + \int d^4x \left( + 2h\rho\Delta\rho^* + 2\Delta^2 + \frac{h^2}{4} \rho A^\mu A_\mu \rho^* + \dots \right),
\end{aligned} \tag{5.3}$$

where the dots stand for spinorial partners and derivative terms that are completely irrelevant for discussing spontaneous symmetry breaking. Also, the  $\rho$ -field does not acquire a non-trivial v.e.v., as already mentioned in the previous section. The scalars that could, in principle, trigger spontaneous symmetry breaking are only  $A$ ,  $A_+$  and  $A_-$ , if one starts from action (5.3).

The only possible way to endow the tensor field  $\lambda_{\mu\nu}$  with a mass, via internal symmetry or supersymmetry breaking, would be by means of a coupling of the form

$$\Sigma^a \Sigma_a \phi^2, \tag{5.4}$$

as dictated by supersymmetry, through the chirality constraints on  $\Sigma$  and  $\phi$ . No matter the number of scalar superfields present in the model, whenever the breaking takes place and a mass is generated for  $\lambda_{\mu\nu}$  as a byproduct, one notices that both the imaginary part of  $\rho$  and one longitudinal degree of freedom of  $\lambda_{\mu\nu}$  provide the spectrum with a tachyonic excitation, without any chance of avoiding this fact at the expenses of the  $\Delta$ -field coupling, enriched by an additional Fayet-Iliopoulos term [12] (actually, a  $\Delta$ -type term does not break supersymmetry whenever it is added to the O' Raifeartaigh model. Moreover, a  $\Delta$ -term never couples to  $\lambda_{\mu\nu}$ , since  $\lambda_{\mu\nu} \lambda_{\mu\nu}^*$  is identically vanishing). Therefore, our final conclusion is that the masslessness of

$\rho$  and  $\lambda_{\mu\nu}$  cannot be avoided in a consistent way, just by invoking internal symmetry or supersymmetry breaking as realised by a set of scalar superfields.

## 6 General Conclusions

The supersymmetrization of the matter tensor field first investigated in Ref.[1] has been worked out here in terms of a spinor chiral superfield,  $\Sigma_a$ , whose kinetic and self-interacting terms have been found in  $N = 1$  - superspace. The gauging of the model reveals some peculiarities, such as the need of gauge fields to appear in the matter self-interactions.

Scalar degrees of freedom that accompany the fermionic partners of  $\lambda_{\mu\nu}$  cannot be the source for spontaneous symmetry or supersymmetry breaking, as it could in principle be thought. The reason is that Lorentz invariance prevents  $\Sigma_a$  from developing a non-trivial vacuum expectation value.

A thorough analysis of the coupling between  $\Sigma_a$  and chiral and anti-chiral scalar superfields indicate that no spontaneous breaking takes place. In components, the scalar  $\rho$  and the tensor  $\lambda_{\mu\nu}$  display a quartic coupling to the physical scalar components of the additional matter superfields, namely,  $\rho^2 A_-^2$  and  $\lambda_{\mu\nu}^2 A_-^2$ , and no non-trivial minimum with  $\langle A_- \rangle \neq 0$  shows up (a non-trivial minimum with non-zero vacuum expectation value restricted to the neutral scalar A is possible, but has no consequence for mass generation). So, spontaneous breaking does not happen to be a possibility for inducing a mass for the tensor  $\lambda_{\mu\nu}$ .

As a next step, the analysis of matter tensor fields in the framework of  $N = 2$  extended supersymmetries and the non-Abelian version of the  $N = 1$  model are to be pursued.

## 7 Conventions

$$\sigma^\mu = (1, \sigma), \quad \bar{\sigma}^\mu = (1, -\sigma), \quad \bar{\sigma}_{ba}^\mu = \sigma_{ab}^\mu, \quad (7.1)$$

where  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (7.2)$$

In addition, the matrices  $\sigma^{\mu\nu}$  and  $\bar{\sigma}^{\mu\nu}$  ( $(\frac{1}{2}, 0)$  and  $(0, \frac{1}{2})$  SO(1,3) generators) are

given by

$$\begin{aligned}\sigma_{a\dot{a}}^\mu \bar{\sigma}^{\nu\dot{a}b} &= \eta^{\mu\nu} \delta_a^b - i(\sigma^{\mu\nu})_a^b \\ \bar{\sigma}^{\mu\dot{a}a} \sigma_{a\dot{b}}^\nu &= \eta^{\mu\nu} \delta_{\dot{b}}^{\dot{a}} - i(\bar{\sigma}^{\mu\nu})_{\dot{b}}^{\dot{a}},\end{aligned}\tag{7.3}$$

and the trace is

$$\sigma_{a\dot{a}}^\mu \bar{\sigma}^{\nu\dot{a}b} \sigma_{b\dot{b}}^\alpha \bar{\sigma}^{\beta\dot{b}a} = 2 \left( \eta^{\mu\nu} \eta^{\alpha\beta} - \eta^{\mu\alpha} \eta^{\nu\beta} + \eta^{\mu\beta} \eta^{\nu\alpha} + i \varepsilon^{\mu\nu\alpha\beta} \right),\tag{7.4}$$

where  $\varepsilon^{0123} = -\varepsilon_{0123} = 1$ .

The summation convention is:

$$\theta\eta = \theta^a \eta_a, \quad \bar{\theta}\bar{\eta} = \bar{\theta}_{\dot{a}} \bar{\eta}^{\dot{a}},\tag{7.5}$$

where lowering and raising of indices are effected through

$$\theta^a = \varepsilon^{ab} \theta_b, \quad \theta_a = \varepsilon_{ab} \theta^b,\tag{7.6}$$

with  $\varepsilon_{ab} = -\varepsilon_{ba}$ , (the same for dotted indices). Differentiation with respect to the anticommuting parameters  $\theta_a$ ,  $\bar{\theta}_{\dot{a}}$  is defined by

$$\frac{\partial}{\partial \theta^a} \theta^b = \delta_a^b, \quad \frac{\partial}{\partial \bar{\theta}^{\dot{a}}} \bar{\theta}^{\dot{b}} = \delta_{\dot{a}}^{\dot{b}}.\tag{7.7}$$

Covariant derivatives with respect to the supersymmetry transformations are:

$$\begin{aligned}D_a &= \frac{\partial}{\partial \theta^a} - i \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu \\ \bar{D}_{\dot{a}} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} + i \theta^a \sigma_{a\dot{a}}^\mu \partial_\mu,\end{aligned}\tag{7.8}$$

and they obey the anticommutation relations

$$\{D_a, \bar{D}_{\dot{a}}\} = 2i \sigma_{a\dot{a}}^\mu \partial_\mu; \quad \{D_a, D_b\} = 0 = \{\bar{D}_{\dot{a}}, \bar{D}_{\dot{b}}\}.\tag{7.9}$$

The Dirac matrices,  $\gamma^\mu$ , playing a role in the purely physical spinorial kinetic term,  $i \bar{\Psi} \gamma^\mu \partial_\mu \Psi$ , have the following expression in terms of the Pauli matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}.\tag{7.10}$$

The spinor  $\bar{\Psi}$  is defined as usually:

$$\bar{\Psi} = \Psi^\dagger \gamma^0, \quad \text{where } \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.\tag{7.11}$$

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